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IMPROVED HYPERSONIC LAMINAR WAKE CALCULATIONS INCLUDING RATE CHEMISTRY

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by

Martin H. Steiger

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IMPROVED HYPERSONIC
LAMINAR WAKE
CALCULATIONS INCLUDING
RATE CHEMISTRY

TECHNICAL REPORT NO. 249

By Martin H. Steiger

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TABLE OF CONTENTS

<u>Section</u>	<u>Title</u>	<u>Page</u>
I	Introduction	1
II	Analysis	3
III	Calculations	8
	References	14
	Symbols	15
	Tables	18
	Figures	27

TECHNICAL REPORT NO. 249

IMPROVED HYPERSONIC LAMINAR WAKE CALCULATIONS

INCLUDING RATE CHEMISTRY

By Martin H. Steiger

I INTRODUCTION

This report is concerned with the prediction of the properties of a laminar, axisymmetric, hypersonic wake which may not be in thermodynamic equilibrium. The flow is treated as a continuum, by a boundary layer approximation, utilizing integral method techniques according to the approach developed in Reference 1.

In Reference 1 Bloom and Steiger, considered both laminar and turbulent wakes. Their integral-method approach involves satisfaction of the conservation equations on the average over a normal section, and satisfaction of the differential equations exactly along the central axis. Reasonable profiles in terms of a transformed normal coordinate and $(i + 4)$ undetermined parameters were assumed, where i denotes the number of components of the mixture considered. These parameters are utilized to express the axial variation of the physical quantities of interest.

The $(i + 4)$ parameters were u_o , H_o , δ_m , a_{O_o} , a_{N_o} , $a_{NO_o^+}$ and the equations utilized in their solution were the momentum integral, momentum boundary condition, energy integral or energy boundary condition, and conservation of species i boundary conditions. The development of the solution was straight-forward, but due to the limitations of a small digital computer used for the computations several approximations in the calculation procedure were made which may have caused some error.

The purpose of this report is to refine the aforementioned work in preparation for large-computer calculation. In particular the present analysis includes:

- (a) An improved rate chemistry and representative thermodynamics.

The air chemistry includes rate processes which involve seven species, i.e., O_2 , O , N_2 , N , NO , NO^+ and e^- . The pertinent mass action laws and the development of the chemical kinetics and the thermodynamics are to be found in Reference 4.

- (b) An additional parameter is introduced (H_{nn_0}) which couples the energy integral equation and boundary condition. This permits a simple evaluation of the effect of an initial defect in the stagnation enthalpy and/or Prandtl - Lewis numbers different from unity on the flow field.

- (c) The system of equations is programmed on the IBM 7090 digital computer. This enables such advantages as automatic interval change, utilization of a Runge-Kutta scheme in the forward integration, test for maximum truncation error, evaluation of the temperature (which is given in terms of a transcendental algebraic expression) to any desirable accuracy and calculation of profiles in the physical plane at any predetermined axial stations.

References 1 and 2 give a detailed discussion of the problems associated with wakes behind blunt bodies traveling at hypersonic speeds and a compilation of the more recent developments and references. Of particular interest at the present time are wakes with so-called "turbulent cores" and three-dimensional effects. The former is discussed in References 1 and 2 and expanded in References 3 and 5, while the latter is considered in Reference 6.

The collaboration of Dr. Martin H. Bloom during the course of this work is acknowledged with thanks. Also, thanks are extended to Gertrude Weilerstein who programmed the analysis.

II ANALYSIS

In the following system of equations for mixtures of perfect gases it is assumed that the binary diffusion coefficients of several species are approximately the same*, so that each diffusion velocity V_i is given by Fick's law $\alpha_i V_i = -D \text{ grad } \alpha_i$. Therefore, a single Lewis number ($Le = \rho D C_p / k$) appears, wherein the fluid properties are those of the mixture. Likewise a single Prandtl number ($Pr = \mu C_p / k$) of the mixture is defined. Both parameters subsequently are assumed constant along the axis.

The equations that govern the flow in an axially-symmetric laminar wake are:

Overall Mass Conservation

$$(\rho u r)_x + (\rho v r)_r = 0 \quad (1)$$

Momentum

$$(\rho u^2 r)_x + (\rho u v r)_r = (\mu r u_r)_r \quad (2a)$$

$$P_r = 0 \quad (2b)$$

Energy

$$\begin{aligned} (\rho u r H)_x + (\rho v r H)_r = & \left[\frac{\mu}{\sigma} r H_r \right]_r + \left[\frac{\mu}{\sigma} (\sigma - 1) r u u_r \right]_r + \\ & + \left[\frac{\mu}{\sigma} (Le - 1) r \sum_i h_i \alpha_{i,r} \right]_r \end{aligned} \quad (3)$$

* For an elaboration of this point see Reference 1, Appendix A.

Mass Conservation of Species i ($i = O_2, N_2, O, N, NO, NO^+$ and e^-):

$$(\rho u r a_i)_x + (\rho v r a_i)_r = \left[\frac{\mu Le}{\sigma} r a_{i,r} \right]_r + \rho r W_i \quad (4)$$

where notation not defined in the text is defined at the end of the report.

The following lists the boundary conditions (those used in exact solutions are enclosed in curved brackets) and auxiliary conditions employed in the integral method. The latter consist of the conservation equations evaluated along the axis, and of outer edge conditions, which, in this instance, express the neglect of radial gradients at the outer edge.

The boundary conditions are:

$$\text{at } r=0: \left\{ v=0, u_r=0, H_r=0, a_{i,r}=0 \right\} \quad (5)$$

$$\rho u u_x = 2(\mu u_r)_r \quad (6)$$

$$\begin{aligned} \rho u H_x = & 2 \left[\frac{\mu}{\sigma} H_r \right]_r + 2 \left[\frac{\mu}{\sigma} (\sigma-1) u u_r \right]_r + \\ & + 2 \left[\frac{\mu}{\sigma} (Le-1) \sum_i h_i a_{i,r} \right]_r \end{aligned} \quad (7)$$

$$\rho u a_{i,x} = 2 \left[\frac{\mu Le}{\sigma} a_{i,r} \right]_r + \rho W_i \quad (8)$$

$$u = u_0(x), \quad H = H_0(x), \quad H_{rr} = H_{rr0}(x), \quad a_i = a_{i0}(x) \quad (9)$$

$$\begin{aligned} \text{as } r \rightarrow \delta \quad u = u_e, \quad H = H_e, \quad a_i = a_{ie} \\ \left\{ \text{where } u_e, a_{ie} \text{ and } H_e \text{ are constant} \right\} \end{aligned} \quad (10)$$

$$u_r = u_{rr} = 0, \quad H_r = H_{rr} = 0, \quad a_{i,r} = a_{i,rr} = 0 \quad (11)$$

Integral equations are derived from Equations (1-4) assuming that integrated transport terms, that is, terms like ru_r , are negligible at the outer edge limit.

The integral equations are;

Momentum

$$\frac{d}{dx} \left[\int_0^\delta \bar{\rho} \bar{u} (1 - \bar{u}) r dr \right] = 0 \quad \text{or}$$

$$\theta = \theta_c \quad \text{where } \theta = \int_0^\delta \bar{\rho} \bar{u} (1 - \bar{u}) r dr \quad (12)$$

Energy

$$\frac{d}{dx} \left[\int_0^\delta \bar{\rho} \bar{u} (1 - \bar{H}) r dr \right] = 0 \quad \text{or}$$

$$\theta_E = \theta_{Ec} \quad \text{where } \theta_E = \int_0^\delta \bar{\rho} \bar{u} (1 - \bar{H}) r dr \quad (13)$$

Species Concentration

$$\frac{d}{dx} \left[\int_0^\delta \bar{\rho} \bar{u} (a_{i,e} - a_i) r dr \right] = - \frac{1}{u_e} \int_0^\delta \bar{\rho} W_i r dr \quad (14)$$

The following transformation are now introduced into the integral equations (12 - 14) and boundary conditions (6-8)

$$mdm = \bar{\rho} r dr, \quad m = \delta_m n \quad (15a)$$

$$\delta^2 / 2 = \int_0^{\delta_m} \frac{mdm}{\bar{\rho}} = \delta_m^2 \int_0^{n=1} \frac{ndn}{\bar{\rho}} \quad (15b)$$

The working forms of the governing equations and boundary conditions are:

$$\delta_m^2 \int_0^1 \bar{u}(1 - \bar{u}) n dn = \theta_c \quad (16)$$

$$u_o u_{ox} = \frac{2\mu_o}{\rho_e \delta_m^2} u_{nn_o} \quad (17)$$

$$\delta_m^2 \int_0^1 \bar{u}(1 - \bar{H}) n dn = \theta_{Ec} \quad (18)$$

$$u_o \frac{dH_o}{dx} = \frac{2\mu_o}{\rho_e \delta_m^2} \left[\frac{H_{nn_o}}{\sigma_o} + \frac{(\sigma_o - 1)}{\sigma_o} u_o u_{nn_o} + \frac{(Le_o - 1)}{\sigma_o} \sum_i h_{i_o} a_{i_{nn_o}} \right] \quad (19)$$

$$\frac{d}{dx} \left[\delta_m^2 \int_0^1 u(a_{ie} - a_i) n dn \right] = -\delta_m^2 \int_0^1 W_i n dn \quad (20)$$

$$u_o \frac{da_{i_o}}{dx} = \frac{2\mu_o}{\rho_e \delta_m^2} \left(\frac{Le_o}{\sigma_o} \right) a_{i_{nn_o}} + W_{i_o} \quad (21)$$

and

$$\text{at } n = 0; \quad u = u_o(x), \quad u_n = 0, \quad H_n = 0 \quad (22a)$$

$$a_{i_n} = 0 \quad H_{nn} = H_{nn_o}(x)$$

$$n = 1; \quad u = u_e, \quad H = H_e, \quad a_i = a_{ie} \quad (22b)$$

$$u_n = u_{nn} = H_n = H_{nn} = a_{i_n} = a_{i_{nn}} = 0$$

A compromise between simplicity and accuracy influences the choice of specific procedures for solutions by the integral method. Here the profiles are depicted by polynomials in n with undetermined parameters (u_o , H_o , H_{nn_o} and a_{i_o}) to express streamwise variation. Another undetermined parameter, the wake thickness

(δ_m) is included implicitly. The equations that will be used to solve for these ($i + 4$) undetermined parameters will be Equations (16 - 19) and (21). Clearly, additional parameters can be included and additional differential or integral conditions (for example, Equation (20)) satisfied at will; however, the more usual approach, underlying simplicity, will be followed here. The profile selection restricts the initial profile to the specified form. In particular cases more accurate curve-fits of the initial conditions in terms of n can be employed. Within reasonable limits these will not have a substantial influence on the overall results. The peak variations represented by u_o , H_o and a_{i_o} are believed to be the most influential parameters.

The assumed profiles, which satisfy the appropriate boundary conditions are:

$$\frac{u - u_o}{u_e - u_o} = \frac{a_i - a_{i_o}}{a_{i_e} - a_{i_o}} = 6n^2 - 8n^3 + 3n^4 \quad (23a)$$

and

$$H = H_o + (H_e - H_o) (10n^3 - 15n^4 + 6n^5) + \frac{H_{nn_o}}{2} (n^2 - 3n^3 + 3n^4 - n^5) \quad (23b)$$

III CALCULATIONS

By utilizing the assumed profiles (23a-b) the governing equations (16-19) and (21) are reduced to a form that only involves the (i + 5) undetermined parameters and variables that depend explicitly on the thermodynamic state. To complete the system, these equations are supplemented by the thermodynamics and chemical kinetics developed in Reference 4.

This system has been programmed on the IBM 7090 digital computer by Gertrude Weilerstein*. The programming techniques in the present case are analogous to those described in Reference 4. Also, the present program incorporates a procedure by which both the normal and axial distribution of the pertinent variables are typed out at predetermined values of the axial coordinate. The calculation procedure is described below.

The input data are, in general, the conditions at the edge of the viscous layer (i.e., u_e , T_e , ρ_e , P_e , H_e and α_{ie}), the initial data at the axis necessary for the forward integration (i.e., θ_c , u_{oc} , H_{oc} , α_{ioc} and H_{nnoc}), the Lewis and Prandtl number (Le_o , σ_o), the initial value of the streamwise coordinate, etc.

At the initial station, say $x = x_c$, sufficient information is known to calculate the following:

(A) Profiles (for $0 \leq n \leq 1$ at intervals of $\Delta n = 0.1$):

$$u = u_o + (u_e - u_o)(6n^2 - 8n^3 + 3n^4) \quad (24a)$$

* A summary of the data preparation for the 7090 digital program has been prepared by Gertrude Weilerstein of GASL

$$H = H_0 + (H_e - H_0)(10n^3 - 15n^4 + 6n^5) + \frac{H_{nnO}}{2}(n^2 - 3n^3 + 3n^4 - n^5) \quad (24b)$$

$$a_i = a_{i0} + (a_{ie} - a_{i0})(6n^2 - 8n^3 + 3n^4)$$

$$i = O_2, N_2, O, N, NO, NO^+, e^- \quad (24c)$$

$$h = H - \frac{u^2}{2} \quad (24d)$$

and for the corresponding values of n , the temperature (T) is calculated from

$$\begin{aligned} h = RT & \left[\sum_J \left(\frac{a_J}{M_J} \right) \left(\Lambda_J + \frac{7}{2} \right) + \frac{5}{2} \sum_k \frac{a_k}{M_k} \right] + a_o \left(\frac{D_{O_2}}{2m_o} \right) + \\ & + a_N \left(\frac{D_{N_2}}{2m_n} \right) + \frac{a_{NO}}{m_{NO}} \left(\frac{D_{N_2} + D_{O_2}}{2} - D_{NO} \right) + \\ & + \frac{a_{NO^+}}{m_{NO^+}} \left(I_{NO} + \frac{D_{N_2} + D_{O_2}}{2} - D_{NO} \right) \end{aligned}$$

where

$$\Lambda_J = \frac{T_j^V / T}{\left(e^{T_j^V / T} - 1 \right)} \quad (24e)$$

and

$$J = O_2, N_2, NO, NO^+ ; \quad k = O, N, e^-$$

the density by:

$$\frac{\rho_e}{\rho} = \frac{T}{T_e} \frac{\sum_i \frac{a_i}{M_i}}{\sum_i \frac{a_{ie}}{M_{ie}}} ; \quad i = O_2, N_2, O, N, NO, NO^+, e^- \quad (24f)$$

the physical thickness by:

$$r = \left[\frac{210 \theta_c u_e^2}{(u_e - u_o)(10u_e + 11u_o)} \right]^{1/2} \left[2 \int_0^n \frac{\rho_e}{\rho} n dn \right]^{1/2} \quad (24g)$$

and the particle concentration of electrons by:

$$N_{e-} = \frac{a_{e-} \eta \rho}{M_{e-}} \quad (24h)$$

Therefore, profiles of the thermodynamic state and all pertinent variables are calculated as functions of both the transformed (n) and physical (r) normal coordinates. This procedure is repeated at predetermined values of the axial coordinate (s).

The calculation of the coefficient of viscosity at the axis (μ_o) completes the necessary information for a forward integration. It is given by

$$\mu_o = 5.48 \times \frac{T_o^{3/2}}{1.8 [T_o + 199]} \times 10^{-8} \quad (25)$$

The axial solution is then obtained from the following system (here we define $s = x/L$ where L is a characteristic length and equals unity for an isobaric flow field, therefore $L = 1$ in this analysis)

$$\frac{du_o}{ds} = \frac{L \mu_o}{\rho_e u_e^2 \theta_c} \frac{4}{35 u_o} (u_e - u_o)^2 (10u_e + 11u_o) \quad (26)$$

$$\begin{aligned} \frac{dH_o}{ds} = & \frac{4}{35} \left[\frac{L \mu_o}{\rho_e \theta_c u_e^2} \right] \left[\frac{u_e - u_o}{u_o} (10u_e + 11u_o) \right] \left[\frac{H_{nn0}}{120} + \right. \\ & \left. + \frac{(\sigma_o - 1)}{\sigma_o} u_o (u_e - u_o) + \frac{(Le_o - 1)}{\sigma_o} \sum_i h_{i0} (a_{i_e} - a_{i_o}) \right] \quad (27a) \end{aligned}$$

where

$$\sum_i h_{i_o} (a_{i_e} - a_{i_o}) = \sum h_{i_o} a_{i_e} - h_o \quad (27b)$$

and assuming that the external flow only consists of the undissociated components of air, it follows that

$$\sum h_{i_o} a_{i_e} = h_{O_{2o}} a_{O_{2e}} + h_{N_{2o}} a_{N_{2e}} \quad (27c)$$

where

$$h_{O_{2o}} = RT_o \frac{a_{O_{2o}}}{M_{O_2}} \left(\frac{7}{2} + \Lambda_{O_{2o}} \right)$$

and

$$h_{N_{2o}} = RT_o \frac{a_{N_{2o}}}{M_{N_2}} \left(\frac{7}{2} + \Lambda_{N_{2o}} \right)$$

$$H_{nn_o} = \frac{4(H_e - H_o)(349 u_e + 311 u_o)}{43 u_e + 23 u_o} - K \left[\frac{4(u_e - u_o)(10 u_e + 11 u_o)}{43 u_e + 23 u_o} \right] \quad (28a)$$

where

$$K = \frac{4(H_e - H_{oc})(349 u_e + 311 u_{oc}) - H_{nn_{oc}}(43 u_e + 23 u_{oc})}{4(u_e - u_{oc})(10 u_e + 11 u_{oc})} \quad (28b)$$

Obviously, Equations (27) and (28) permit an estimate as to the effect that initial defects in stagnation enthalpy and/or Lewis-Prandtl number has on the flow field. For a flow which is initially isoenergetic, that is, $H_e = H_{oc}$ and $H_{nn_o} = 0$, it follows that $K = 0$. If it is further assumed that the Prandtl and Lewis numbers are unity then Equations (27) and (28) state that the flow remains isoenergetic (i.e., $H = H_{oc} = H_e = \text{constant throughout } s \geq s_c$). Clearly, under these conditions the governing partial differential equation yields the same results.

$$u_o \frac{da_{i_o}}{ds} = \frac{L\mu_o}{\rho_e u_e^2 \theta_c} \frac{4}{35} \frac{Le_o}{\sigma_o} (u_e - u_o)(10u_e + 11u_o)(a_{i_e} - a_{i_o}) + \frac{M_i}{\rho} \left(\sum_l \dot{W}_l \right)_{i_o} \quad (29)$$

where $i = O_2, N_2, O, N, NO, NO^+, e^-$.

The term $\frac{M_i}{\rho} \left(\sum_l \dot{W}_l \right)_{i_o}$ in Equation (29) is the net rate of production of i^{th} species (1/sec) - with contribution from l mass action laws - and depends on the thermochemistry of air. In Reference 4 the principles of chemical kinetics are applied to the following reactions:



Equations (9a - k) of Reference 4 are identical to the expression $\frac{M_i}{\rho} \left(\sum_l \dot{W}_l \right)_{i_o}$ for $i = O_2, N_2, O, N, NO, NO^+, e^-$ and therefore will not be reproduced here.

Example Calculation

Calculations have been made of the laminar wake properties for one trajectory point of a blunt body, i.e., $u_{\infty} = 18,900$ ft/sec at 150,000 ft. This corresponds to Case 1 of Reference 1. The initial data is shown in Table 1 and the axial distribution of the pertinent variables in Figure 1a-d. Several typical type-outs, including profiles, are given in Tables (2a-f). The computation time on the 7090 digital computer was approximately five minutes.

1. Bloom, M.H. and Steiger, M.H., Hypersonic Axisymmetric Wakes Including Effects of Rate Chemistry, General Applied Science Laboratories, Inc., Technical Report No. 180.
2. Vaglio-Laurin, R. and Bloom, M.H., Chemical Effects in External Hypersonic Flows, International Hypersonics Conference, held at Massachusetts Institute of Technology, August 1961, Reprint No. 1976-61.
3. Bloom, M.H., and Steiger, M.H., Turbulent Wakes Behind Blunt Bodies Traveling at Hypersonic Speeds, General Applied Science Laboratories, Inc. Technical Report (in preparation)
4. Steiger, M.H., Improved Rate Chemistry Program for One-Dimensional Inviscid Air Flow With Prescribed Pressure Variation, General Applied Science Laboratories, Inc., Technical Report No. 246, August 1961.
5. Hromas, L. and Lees, L., Turbulent Diffusion in the Wake of a Blunt-Nosed Body at Hypersonic Speeds, Space Technology Laboratories, (Memo in preparation, made available in preliminary form June 1961).
6. Steiger, M.H. and Bloom, M.H., Three-Dimensional Effects in Viscous Wakes, Polytechnic Institute of Brooklyn, PIBAL Report 711 (to be issued).

SYMBOLS

c_p	specific heat, constant pressure
D_i	dissociation energy per mole, (see Reference 4)
D_{iJ}	binary diffusion coefficient
e^-	refer to electron species
h	static enthalpy
H	stagnation enthalpy
H_{nn_0}	curvature of the stagnation enthalpy profile at $n = 0$
I_J	ionization energy per mole (see Reference 4)
k	thermal conductivity
K	defined by Equation (28)
L	characteristic length, this report $L = 1$ ft.
Le	Lewis number $Le = \frac{\rho D_{iJ} \cdot c_p}{k}$
m	transformed normal coordinate Equation (15a, b)
M_i	molecular weight, mass/mole
n	transformed normal coordinate, Equation (15 a, b)
N	refers to atomic nitrogen
N_2	refers to molecular nitrogen
N_e^-	refers to particle concentration of electrons, electrons/cc
O	refers to atomic oxygen
O_2	refers to molecular oxygen
NO	refers to nitric oxide
NO^+	refers to ionized nitric oxide
r	normal coordinate
R	gas constant per mole

Symbols (Continued)

16

s	transformed streamwise coordinate	$s = x/L$
T	temperature, °K	
T_i^V	characteristic vibrational temperature	
u, v	streamwise and normal velocity component	
u_∞	free stream flight velocity	
$\frac{M_i}{\rho} \left(\sum_i W_i \right)_i$	W_i	net rate of production of species i
x	streamwise coordinate	
X	catalyst	
α_i	mass fraction of species i	
δ	physical viscous layer thickness	
δ_m	transformed viscous layer thickness	
θ	momentum thickness	$\theta = \int_0^\delta \bar{\rho} \bar{u} (1 - \bar{H}) r dr$
θ	energy thickness	$\theta_E = \int_0^\delta \bar{\rho} \bar{u} (1 - \bar{H}) r dr$
θ_i	i th species thickness	$\theta = \int_0^\delta \bar{\rho} \bar{u} (\alpha_{Le} - \alpha_i) r dr$
μ	coefficient of viscosity	
ρ	density	
Λ	see Equation (24c)	
Σ	summation	
η	Avagadro's number	
σ	Prandtl number	$\sigma = \frac{\mu c_p}{k}$

c	conditions at initial station
e	conditions at edge of viscous layer, constant
e^-	refers to electron species
N	refers to atomic nitrogen
N_2	refers to molecular nitrogen
NO	refers to nitric oxide
NO^+	refers to ionized nitric oxide
O	refers to atomic oxygen
O_2	refers to molecular oxygen
i, J, k	indices, (i = O_2 , N_2 , O, N, NO , NO , e^-)
o	values evaluated along axis
x, y, n	denotes partial differentiation with respect to indicated variable
∞	undisturbed flight conditions

Superscripts

-	denotes nondimensional quantities with respect to condition
	at the edge of the viscous layer, $\bar{u} = \frac{u}{u_e}$,
\bar{H}	$= \frac{H}{H_e}$, etc.

TABLE IA. Flight Conditions

(1)	Velocity	18,900 ft/sec
(2)	Pressure	2.94 #/sq. ft
(3)	Density	3.39×10^{-6} slug/ft ³
(4)	Temperature	280 °K

B. Initial Conditions

(1) Mass Fractions

a_O	=	.232	a_{NO}	=	0
a_{O_2}	=	0	a_{NO^+}	=	5.63×10^{-4}
a_N	=	.133	a_e	=	1.029×10^{-8}
a_{N_2}	=	.635			

(2)	Velocity	13,078.8 ft/sec
(3)	θ_c	.0065 ft ²
(4)	H_{nn_O}	0
(5)	Stagnation enthalpy	1.81646×10^8 ft ² /sec ²

C. Miscellaneous

- (1) Crocco Integral $H = \text{constant throughout}$
- (2) $Le_O = \sigma_O = 1$

TABLE 2 - COMPUTATION

PARAMETERS FOR CONDITIONS AT EDGE OF VISCOUS LAYER

PRESSURE= 2.9400000E 00 LB/FT SQ DENSITY= 3.3900000E-06 SLUG/CU FT TEMPERATURE= 2.8000000E 02 DEGREES KELVIN
 VELOCITY= 1.8900000E 04 FT/SEC TOTAL ENTHALPY= 1.8164608E 08 FT SQ/SEC SQ
 ALPHA(0) = 0.
 ALPHA(N0) = 0.
 ALPHA(02) = 2.3200000E-01 ALPHA(N) = 0.
 ALPHA(N0+) = 0. ALPHA(E) = 0.
 ALPHA(N2) = 7.6799999E-01

PARAMETERS FOR INITIAL CONDITIONS AT S=0

L=1.0 H(N0)= 0.
 VELOCITY= 1.3078800E 04 FT/SEC THETA(C)= 6.4999999E-03 L(E0)= 1.0000000E 00 P(R0)= 1.0000000E 02
 ALPHA(0) = 2.3200000E-01 TOTAL ENTHALPY= 1.8164608E 08 FT SQ/SEC SQ
 ALPHA(N0+) = 0.
 ALPHA(02) = 0.
 ALPHA(N) = 1.3300000E-01 ALPHA(N2) = 6.3500000E-01
 ALPHA(E) = 1.0290553E-08

PARAMETERS FOR INITIAL CONDITIONS AT START OF INTEGRATION

INITIAL VALUE OF S = 0.
 LIMITING VALUE OF S = 1.0000000E 03 INITIAL STEP SIZE OF S FOR INTEGRATION = 9.9999999E-04
 VELOCITY= 1.3078800E 04 FT/SEC VALUE OF STEP SIZE BETWEEN PRINT OUT OF S = 2.0000000E 01
 ALPHA(02) = 0.
 ALPHA(N0+) = 5.6299999E-04 TOTAL ENTHALPY= 1.8164608E 08 FT SQ/SEC SQ
 ALPHA(N) = 1.3300000E-01 ALPHA(N2) = 6.3500000E-01
 ALPHA(E) = 1.0290553E-08

DIMENSIONS OF OUTPUT

TOTAL ENTHALPY.....FT SQ/SEC SQ VELOCITY.....FT/SEC STATIC ENTHALPY.....FT SQ/SEC SQ
 DENSITY.....SLUG/CU FT TEMPERATURE.....DEGREES KELVIN PARTICLES OF ELECTRONS.PART/CC
 PHYSICAL THICKNESS.....FT

TABLE 2b

STEP	2	S= 2.0473996E 01	H1NN0)= 0.	MU= 7.4170520E-07			
N	PHY THICKNESS	TOTAL ENTHALPY	VELOCITY	STATIC ENTHALPY	DENSITY	TEMPERATURE	PART CF ELEC
0.	0.	1.8164608E 08	1.3251400E 04	9.3846281E 07	8.0043803E-07	8.6852022E 02	4.4103252E 09
0.1	1.0571165E-01	1.8164608E 08	1.3546822E 04	8.9867891E 07	7.8221983E-07	9.2129542E 02	4.0845349E 09
0.2	2.1262335E-01	1.8164608E 08	1.4272667E 04	7.9791571E 07	7.6480622E-07	9.7407277E 02	3.4521070E 09
0.3	3.1803192E-01	1.8164608E 08	1.5218907E 04	6.5840033E 07	7.9281506E-07	9.8289996E 02	2.8465364E 09
0.4	4.1633257E-01	1.8164608E 08	1.6215785E 04	5.0170237E 07	8.9398060E-07	9.1609211E 02	2.3407091E 09
0.5	5.0212390E-01	1.8164608E 08	1.7134812E 04	3.4845186E 07	1.1013615E-06	7.8024892E 02	1.8963639E 09
0.6	5.7227497E-01	1.8164608E 08	1.7887770E 04	2.1659914E 07	1.4696850E-06	6.0931412E 02	1.4511264E 09
0.7	6.2684433E-01	1.8164608E 08	1.8427212E 04	1.1865008E 07	2.0568462E-06	4.4890969E 02	9.4857172E 08
0.8	6.6917431E-01	1.8164608E 08	1.8746358E 04	5.9331140E 06	2.7847810E-06	3.3777817E 02	4.1735252E 08
0.9	7.0434335E-01	1.8164608E 08	1.8879100E 04	3.4358660E 06	3.2919193E-06	2.8798586E 02	6.7110411E 07
1.0	7.3924563E-01	1.8164608E 08	1.8900000E 04	3.0410800E 06	3.3905055E-06	2.7995825E 02	0.

N	ALPHA(0)	ALPHA(021)	ALPHA(02)	ALPHA(01)	ALPHA(04)	ALPHA(04+)	ALPHA(01)
0.	2.3059329E-01	1.6343220E-03	1.2258456E-01	6.4561470E-01	1.188896E-04	5.3211800E-07	9.7267634E-12
0.1	2.1853327E-01	1.3682446E-02	1.1617339E-01	6.5201545E-01	1.1267107E-04	5.0428823E-07	9.2152431E-12
0.2	1.8890203E-01	4.3284435E-02	1.0042128E-01	6.6774195E-01	9.7393835E-05	4.3591107E-07	7.9683294E-12
0.3	1.5027765E-01	8.1870683E-02	7.9888361E-02	6.8824149E-01	7.7479934E-05	3.4678131E-07	6.3390021E-12
0.4	1.0957794E-01	1.2253022E-01	5.8252188E-02	7.0984249E-01	5.6496035E-05	2.52866249E-07	4.6222532E-12
0.5	7.2060408E-02	1.6001072E-01	3.8307678E-02	7.2975459E-01	3.7152800E-05	1.6628688E-07	3.03951752E-12
0.6	4.1322324E-02	1.9071846E-01	2.1967158E-02	7.4606854E-01	2.1304904E-05	9.5355560E-08	1.7433722E-12
0.7	1.9300658E-02	2.1271839E-01	1.0260328E-02	7.5775634E-01	9.9510052E-06	4.4538275E-08	8.1414630E-13
0.8	6.2721390E-03	2.2573405E-01	3.3343015E-03	7.6467112E-01	3.2337812E-06	1.4473613E-08	2.6457552E-13
0.9	8.5318834E-04	2.3114765E-01	4.5355875E-04	7.6754717E-01	4.3988529E-07	1.9688215E-09	3.5989441E-14
1.0	0.	2.3200000E-01	0.	7.6799999E-01	0.	0.	0.

TABLE 2c

STEP	N	S= 8.1913937E 01	H(MNO)= 0.	MU= 9.3232234E-07					
		PHY THICKNESS	TOTAL ENTHALPY	VELOCITY	STATIC ENTHALPY	DENSITY	TEMPERATURE	PART OF ELEC	
0.	0.	0.	1.8164608E 08	1.3794843E 04	8.6497234E 07	5.8050182E-07	1.257427E 03	1.2049521E 09	
0.1	0.1	1.2733121E-01	1.8164608E 08	1.4061842E 04	8.2778373E 07	5.8607040E-07	1.260725E 03	1.1529873E 09	
0.2	0.2	2.5202411E-01	1.8164608E 08	1.4717855E 04	7.3338448E 07	6.1127029E-07	1.2461929E 03	1.0394160E 09	
0.3	0.3	3.7000211E-01	1.8164608E 08	1.5572969E 04	6.0387399E 07	6.7397569E-07	1.1778050E 03	9.117129E 08	
0.4	0.4	4.7632023E-01	1.8164608E 08	1.6474029E 04	4.5949264E 07	7.9709466E-07	1.0420835E 03	7.8623530E 08	
0.5	0.5	5.6683789E-01	1.8164608E 08	1.7304638E 04	3.1920830E 07	1.0164723E-06	8.5368679E 02	6.5934359E 08	
0.6	0.6	6.3953657E-01	1.8164608E 08	1.7985155E 04	1.9913172E 07	1.3918656E-06	6.4713013E 02	5.1772783E 08	
0.7	0.7	6.9525163E-01	1.8164608E 08	1.8472698E 04	1.1025790E 07	1.9910687E-06	4.6503752E 02	3.4592177E 08	
0.8	0.8	7.3786731E-01	1.8164608E 08	1.8761140E 04	5.6558999E 06	2.7475808E-06	3.4266796E 02	1.5512441E 08	
0.9	0.9	7.7340034E-01	1.8164608E 08	1.8881111E 04	3.3979040E 06	3.2850019E-06	2.8862884E 02	2.5229023E 07	
1.0	1.0	8.0759324E-01	1.8164608E 08	1.8900000E 04	3.0410800E 06	3.3905055E-06	2.7995825E 02	0.	
	N	ALPHA101	ALPHA1021	ALPHA1N1	ALPHA1N21	ALPHA1N01	ALPHA1N0+1	ALPHA1E1	
0.	0.	2.3116609E-01	9.8246590E-04	8.8854264E-02	6.7927578E-01	2.1494215E-04	2.0043641E-07	3.6643805E-12	
0.1	0.1	2.1907610E-01	1.3064682E-02	8.4207186E-02	6.8391605E-01	2.0370067E-04	1.8995359E-07	3.4727334E-12	
0.2	0.2	1.8937126E-01	4.2750434E-02	7.2789413E-02	6.9531711E-01	1.7608061E-04	1.6419751E-07	3.0018635E-12	
0.3	0.3	1.5065094E-01	8.1445869E-02	5.7906324E-02	7.1017842E-01	1.4007780E-04	1.3062441E-07	2.3880758E-12	
0.4	0.4	1.0985013E-01	1.2222046E-01	4.2223548E-02	7.2583824E-01	1.0214051E-04	9.5247387E-08	1.7413137E-12	
0.5	0.5	7.2239404E-02	1.5980701E-01	2.7766958E-02	7.4027367E-01	6.7169423E-05	6.2636381E-08	1.1451170E-12	
0.6	0.6	4.1424969E-02	1.9060165E-01	1.5922687E-02	7.5210062E-01	3.8517639E-05	3.5918210E-08	6.5665709E-13	
0.7	0.7	1.9348601E-02	2.1266383E-01	7.4371015E-03	7.6057378E-01	1.7990656E-05	1.6776527E-08	3.0670855E-13	
0.8	0.8	6.2877200E-03	2.2571632E-01	2.4168370E-03	7.6558669E-01	5.8464283E-06	5.4518718E-09	9.9671193E-14	
0.9	0.9	8.5530802E-04	2.3114524E-01	3.2875780E-04	7.6767172E-01	7.9528036E-07	7.4160944E-10	1.3559111E-14	
1.0	1.0	0.	2.3200000E-01	0.	7.6799999E-01	0.	0.	0.	

TABLE 2 d

STEP	10	S = 1.8431376E 02	H(NN0) = 0.	MU = 1.1144444E-06			
N	PHY THICKNESS	TOTAL ENTHALPY	VELOCITY	STATIC ENTHALPY	DENSITY	TEMPERATURE	PART OF ELEC
0.	0.	1.8164608E 08	1.4611056E 04	7.4904596E 07	4.5240979E-07	1.6763062E 03	5.8024594E 08
0.1	1.5378478E-01	1.8164608E 08	1.4833368E 04	7.1602008E 07	4.6596832E-07	1.6448236E 03	5.6637933F 08
0.2	3.0122093E-01	1.8164608E 08	1.5386497E 04	6.3273931E 07	5.0743209E-07	1.5509107E 03	5.3314825C 08
0.3	4.3657628E-01	1.8164608E 08	1.6104895E 04	5.1922253E 07	5.8585795E-07	1.3919373E 03	4.8968879F 08
0.4	5.5515978E-01	1.8164608E 08	1.6861894E 04	3.9484350E 07	7.2057861E-07	1.1765902E 03	4.3917533E 08
0.5	6.5387564E-01	1.8164608E 08	1.7559705E 04	2.7474460E 07	9.4710152E-07	9.2915514E 02	3.7960043E 08
0.6	7.3181024E-01	1.8164608E 08	1.8131421E 04	1.7271864E 07	1.3273110E-06	6.8427588E 02	3.6505376E 08
0.7	7.9069844E-01	1.8164608E 08	1.8541015E 04	9.7614559E 06	1.9361240E-06	4.8014574E 02	2.0784450E 08
0.8	8.3516636E-01	1.8164608E 08	1.8783340E 04	5.2391400E 06	2.7162861E-06	3.4707235E 02	9.4759939E 07
0.9	8.7189728E-01	1.8164608E 08	1.8884131E 04	3.3408820E 06	3.2791423E-06	2.8919670E 02	1.5561040E 07
1.0	9.0716773E-01	1.8164608E 08	1.8900000E 04	3.0410800E 06	3.3905055E-06	2.7995825E 02	0.

N	ALPHA(10)	ALPHA(102)	ALPHA(N)	ALPHA(N2)	ALPHA(N0)	ALPHA(N0+)	ALPHA(E)
0.	2.3068210E-01	1.2395867E-03	4.2013169E-02	7.2591827E-01	5.6154516E-04	1.2383883E-07	2.2641985E-12
0.1	2.1861743E-01	1.3308356E-02	3.9815880E-02	7.2811914E-01	5.3217635E-04	1.1736206E-07	2.1457209E-12
0.2	1.8897478E-01	4.2961068E-02	3.4417188E-02	7.3352664E-01	4.6001779E-04	1.0144877E-07	1.8548314E-12
0.3	1.5033553E-01	8.1613436E-02	2.7379982E-02	7.4057533E-01	3.6595898E-04	8.0705769E-08	1.4755792E-12
0.4	1.0962014E-01	1.2234264E-01	1.9964659E-02	7.4800276E-01	2.6684627E-04	5.8848217E-08	1.0759472E-12
0.5	7.2088159E-02	1.5988737E-01	1.3129116E-02	7.5484945E-01	1.7548287E-04	3.8699637E-08	7.0756205E-13
0.6	4.1338239E-02	1.9064772E-01	7.5287609E-03	7.6045895E-01	1.0062890E-04	2.2191922E-08	4.0574445E-13
0.7	1.9308092E-02	2.1268535E-01	3.5165022E-03	7.6447775E-01	4.7001333E-05	1.0365310E-08	1.8951342E-13
0.8	6.2745548E-03	2.2572331E-01	1.1427589E-03	7.6685537E-01	1.5274032E-05	3.3684184E-09	6.1586234E-14
0.9	8.5351616E-04	2.3114619E-01	1.5544752E-04	7.6784430E-01	2.0777006E-06	4.5820059E-10	8.3774675E-15
1.0	0.	2.3200000E-01	0.	7.6799999E-01	0.	0.	0.

TABLE 2c

STEP	N	16	S= 3.0719336E 02	H(NNO)= 0.	MU= 1.2160996E-06	PHY THICKNESS	TOTAL ENTHALPY	VELOCITY	STATIC ENTHALPY	DENSITY	TEMPERATURE	PART OF ELEC
0.						C.						
0.1						1.7604986E-01	1.8164608E 08	1.5362315E 04	6.3645711E 07	4.0543029E-07	1.9391573E 03	4.2289822E 08
0.2						3.4327505E-01	1.8164608E 08	1.5547336E 04	6.0786245E 07	4.2111437E-07	1.8838548E 03	4.1628494E 08
0.3						4.9467807E-01	1.8164608E 08	1.6001929E 04	5.3615217E 07	4.6738331E-07	1.7360179E 03	3.9937677E 08
0.4						6.2549970E-01	1.8164608E 08	1.6594491E 04	4.3957516E 07	5.5134612E-07	1.5166699E 03	3.7479320E 08
0.5						7.3316627E-01	1.8164608E 08	1.7218892E 04	3.3400958E 07	6.9125438E-07	1.2499979E 03	3.4263690E 08
0.6						8.1744391E-01	1.8164608E 08	1.7794473E 04	2.3324440E 07	9.2229406E-07	9.6654751E 02	3.0063470E 08
0.7						8.8073323E-01	1.8164608E 08	1.8266046E 04	1.4821852E 07	1.3068309E-06	7.0031714E 02	2.4427394E 08
0.8						9.2832366E-01	1.8164608E 08	1.8603896E 04	8.5936120E 06	1.9213184E-06	4.8560847E 02	1.6774301E 08
0.9						9.6755468E-01	1.8164608E 08	1.8803775E 04	4.8551060E 06	2.7091375E-06	3.4840496E 02	7.6863432E 07
1.0						1.0052327E 00	1.8164608E 08	1.8886911E 04	3.2883840E 06	3.2779288E-06	2.8935113E 02	1.2550615E 07
								1.8900000E 04	3.0410800E 06	3.3905055E-06	2.7995825E 02	0.
N												
0.						ALPHA(0)	ALPHA(02)	ALPHA(01)	ALPHA(02)	ALPHA(01)	ALPHA(01)	ALPHA(01)
0.1						2.2486548E-01	5.8059574E-03	4.1700652E-03	7.6266677E-01	2.8338668E-03	1.0071510E-07	1.5414250E-12
0.2						2.1310501E-01	1.7635811E-02	3.9519708E-01	7.6294569E-01	2.6856556E-03	9.5447701E-08	1.7451195E-12
0.3						1.8420980E-01	4.6701757E-02	3.4161174E-03	7.6363102E-01	2.3215037E-03	8.2505810E-08	1.5084954E-12
0.4						1.4654483E-01	8.4589274E-02	2.7176315E-03	7.6452433E-01	1.8468311E-03	6.5636032E-08	1.2005567E-12
0.5						1.0685608E-01	1.2451254E-01	1.9816151E-03	7.6546565E-01	1.3466536E-03	4.7859818E-08	8.7504522E-13
0.6						7.0270466E-02	1.6131433E-01	1.3031454E-03	7.6633336E-01	8.8558341E-04	3.1473470E-08	5.7544534E-13
0.7						4.0295901E-02	1.9146600E-01	7.4727580E-04	7.6704428E-01	5.0782901E-04	1.8048149E-08	3.2995342E-13
0.8						1.8821239E-02	2.1306755E-01	3.4903444E-04	7.6755360E-01	2.3719465E-04	8.4298540E-09	1.5412727E-13
0.9						6.1163436E-03	2.2584751E-01	1.1342583E-04	7.6785493E-01	7.7081203E-05	2.7394522E-09	5.0086779E-14
1.0						8.3199515E-04	2.3116308E-01	1.5429105E-05	7.6798026E-01	1.0485237E-05	3.7264325E-10	6.8132349E-15
						0.	2.3200000E-01	0.	7.6799999E-01	0.	0.	0.

TABLE 2g

STEP	50	S = 1.0004340E 03	H(NN01) = 0.	MU = 9.8597881E-07									
N	PHY THICKNESS	TOTAL ENTHALPY	VELOCITY	STATIC ENTHALPY	DENSITY	TEMPERATURE	PART OF ELIC						
0.	0.	1.8164608E 08	1.7013829E 04	3.6910886E 07	6.2227910E-07	1.3739893E 03	2.0113731E 08						
0.1	1.8978921E-01	1.8164608E 08	1.7112476E 04	3.5227666E 07	6.4679759E-07	1.3288013E 03	1.9812809E 08						
0.2	3.7010485E-01	1.8164608E 08	1.7354849E 04	3.1050692E 07	7.1753719E-07	1.2133525E 03	1.8999455E 08						
0.3	5.3384699E-01	1.8164608E 08	1.7670782E 04	2.5517804E 07	8.4127729E-07	1.0527031E 03	1.7721222E 08						
0.4	6.7666391E-01	1.8164608E 08	1.8003692E 04	1.9579624E 07	1.0377572E-06	8.6916211E 02	1.5939657E 08						
0.5	7.9659712E-01	1.8164608E 08	1.8310571E 04	1.4007566E 07	1.3405909E-06	6.8448075E 02	1.3541074E 08						
0.6	8.9418443E-01	1.8164608E 08	1.8561998E 04	9.3721960E 06	1.7896973E-06	5.2010101E 02	1.0365316E 08						
0.7	9.7267853E-01	1.8164608E 08	1.8742127E 04	6.0124100E 06	2.3831448E-06	3.9465803E 02	6.4473709E 07						
0.8	1.0378338E 00	1.8164608E 08	1.8848696E 04	4.0094100E 06	2.9790674E-06	3.1767127E 02	2.6191254E 07						
0.9	1.0966629E 00	1.8164608E 08	1.8893021E 04	3.1729600E 06	3.3278624E-06	2.8511192E 02	3.9793796E 06						
1.0	1.1551583E 00	1.8164608E 08	1.8900000E 04	3.0410800E 06	3.3905055E-06	2.7995825E 02	0.						
N	ALPHA(10)	ALPHA(1021)	ALPHA(1N)	ALPHA(N2)	ALPHA(N01)	ALPHA(N0+1)	ALPHA(E1)						
0.	1.2201345E-01	1.0872027E-01	5.0316869E-09	7.6689145E-01	2.5572038E-03	3.1202443E-08	5.7061234E-13						
0.1	1.1563215E-01	1.1516780E-01	4.7685297E-09	7.6694942E-01	2.4234620E-03	2.9570555E-08	5.4076931E-13						
0.2	9.9953420E-02	1.3100925E-01	4.1219579E-09	7.6709187E-01	2.0948613E-03	2.5561041E-08	4.6744562E-13						
0.3	7.9516168E-02	1.5165860E-01	3.2791504E-09	7.6727755E-01	1.6665297E-03	2.0334632E-08	3.7186806E-13						
0.4	5.7980795E-02	1.7341747E-01	2.3910578E-09	7.6747321E-01	1.2151833E-03	1.4827402E-08	2.7115500E-13						
0.5	3.8129205E-02	1.9347508E-01	1.5724022E-09	7.6765357E-01	7.9912621E-04	9.7507639E-09	1.7831636E-13						
0.6	2.1864814E-02	2.0990827E-01	9.0167844E-10	7.6780134E-01	4.5825098E-04	5.5914786E-09	1.0225375E-13						
0.7	1.0212526E-02	2.2168148E-01	4.2115217E-10	7.6790721E-01	2.1403795E-04	2.6116444E-09	4.7760251E-14						
0.8	3.3187670E-03	2.2864679E-01	1.3686197E-10	7.6796983E-01	6.9555972E-05	8.4870688E-10	1.5520666E-14						
0.9	4.5144651E-04	2.3154387E-01	1.8617108E-11	7.6799589E-01	9.4615679E-06	1.1544809E-10	2.1112466E-15						
1.0	0.	2.3200000E-01	0.	7.6799999E-01	0.	0.	0.						

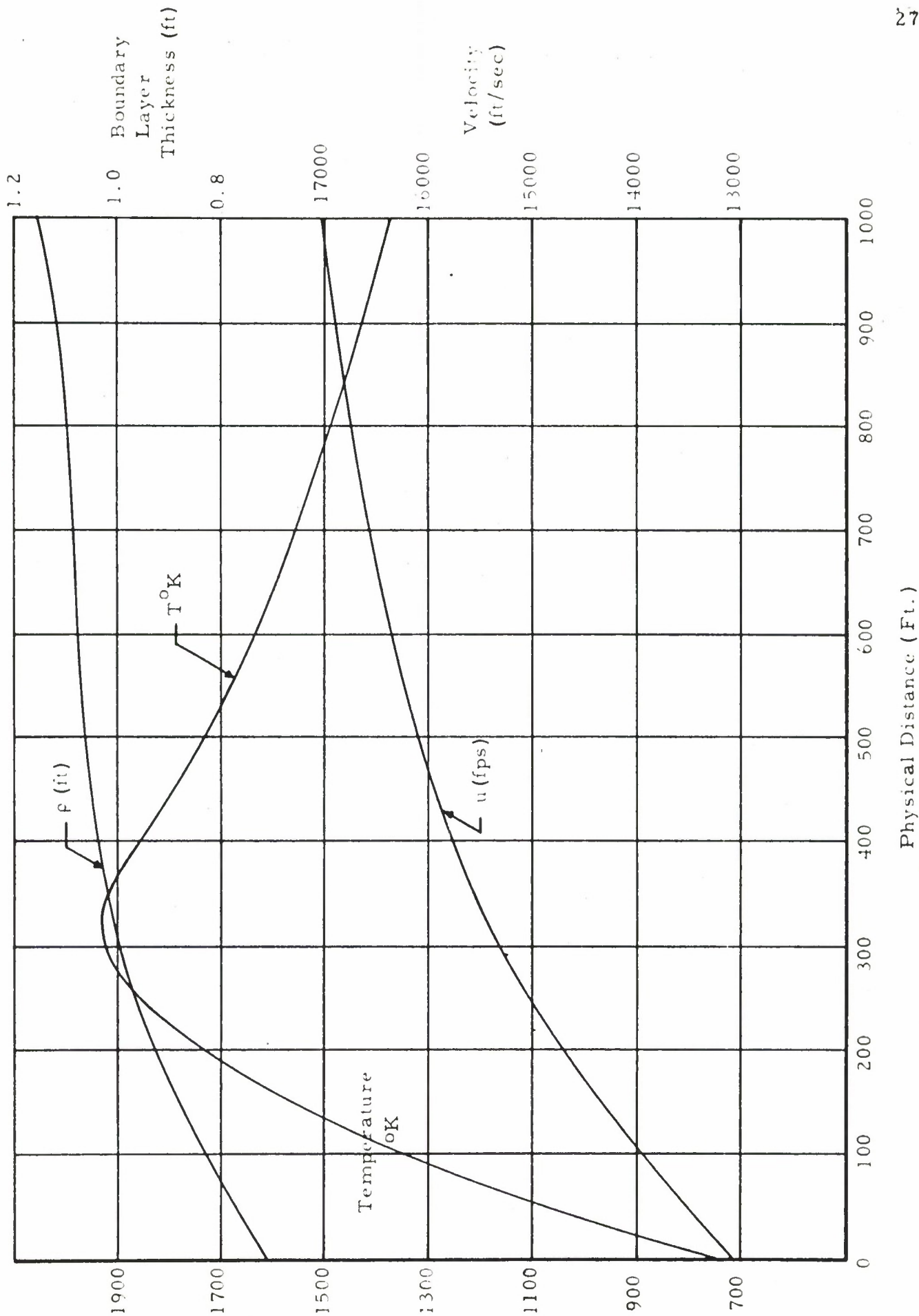


Figure 1a. Distribution of Temperature, Velocity and Physical Boundary Layer Thickness Along the Axis.

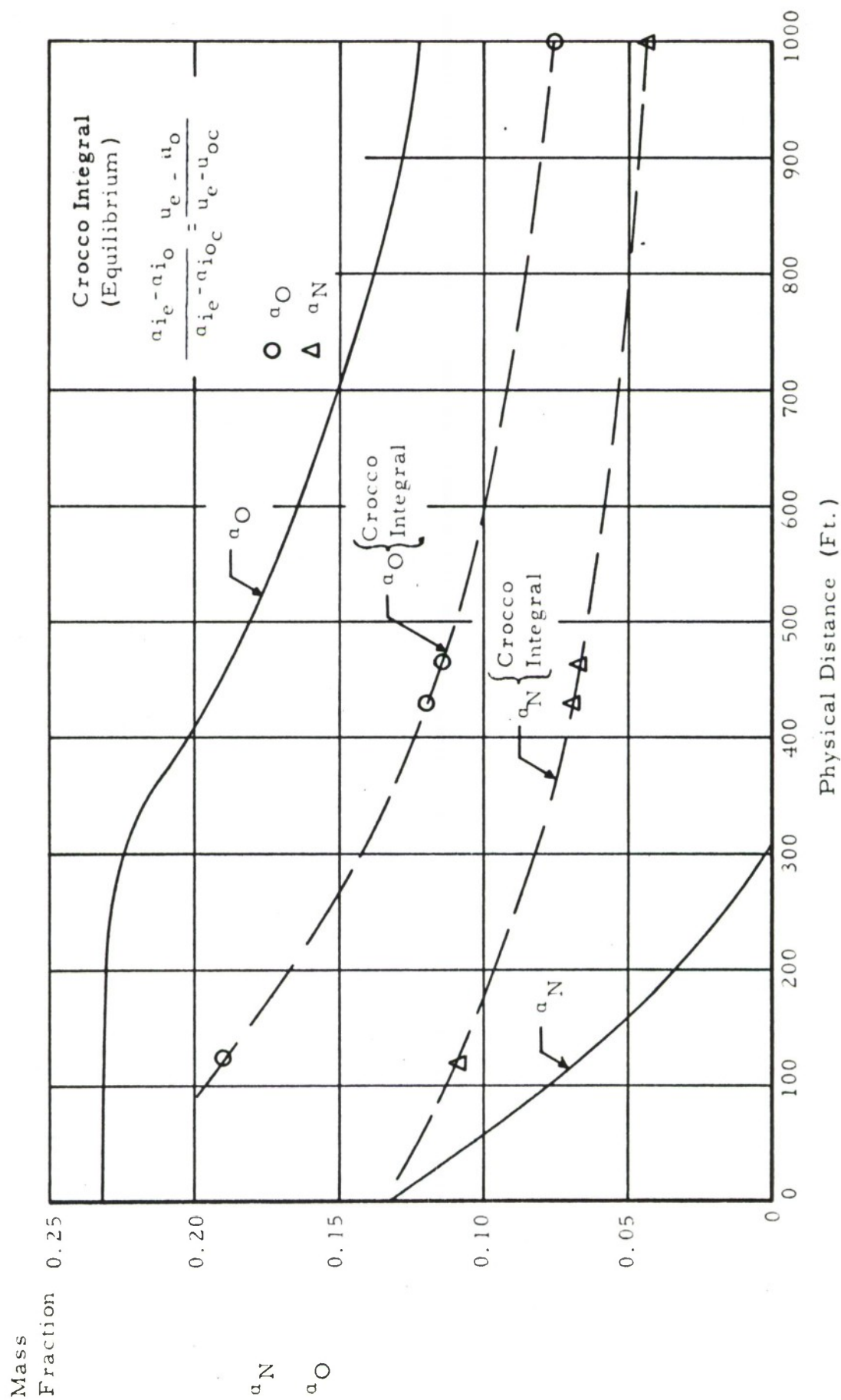


Figure 1b. Distribution of a_O and a_N Along the Axis

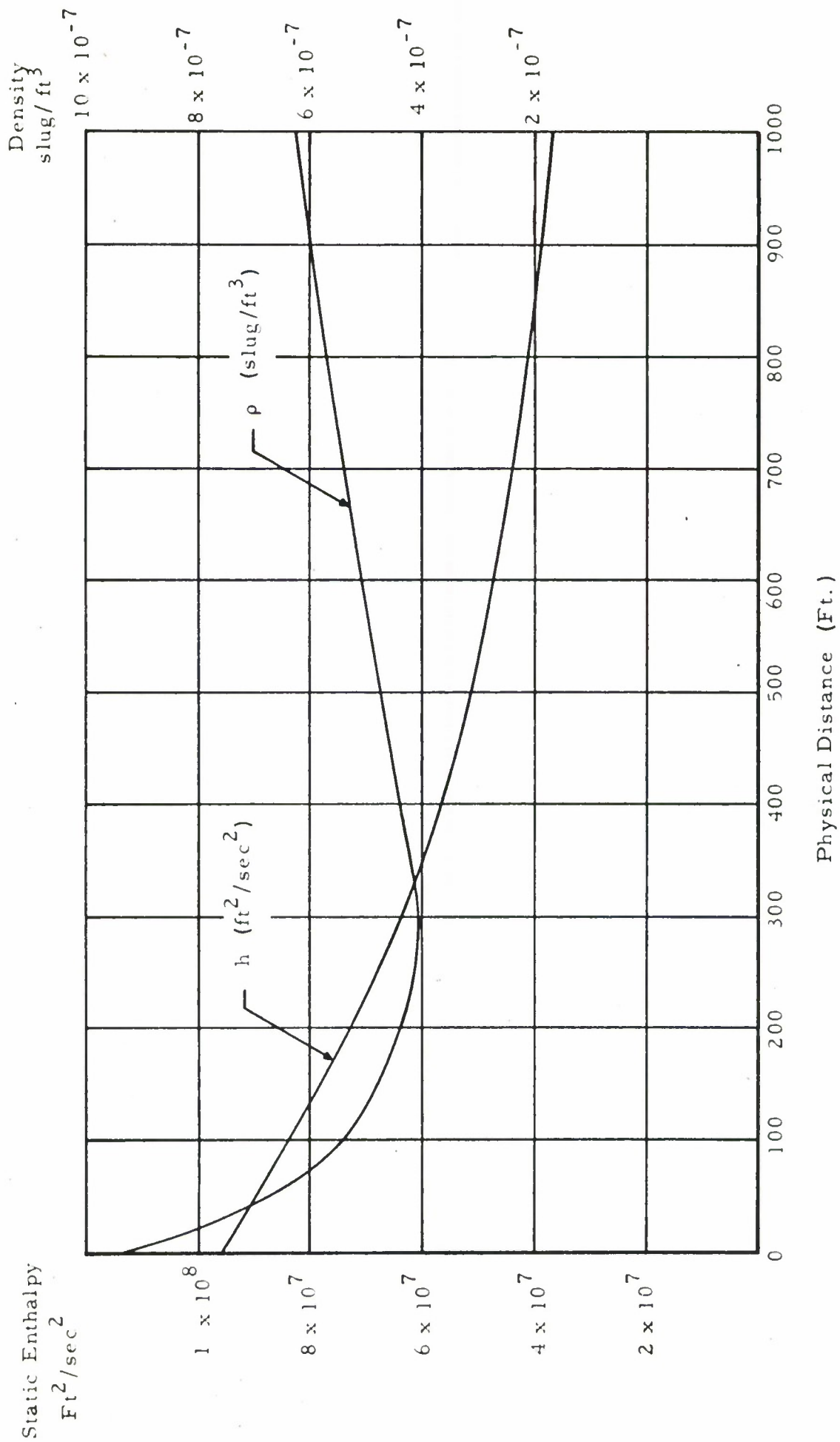


Figure 1 c. Distribution of Enthalpy and Density Along the Axis

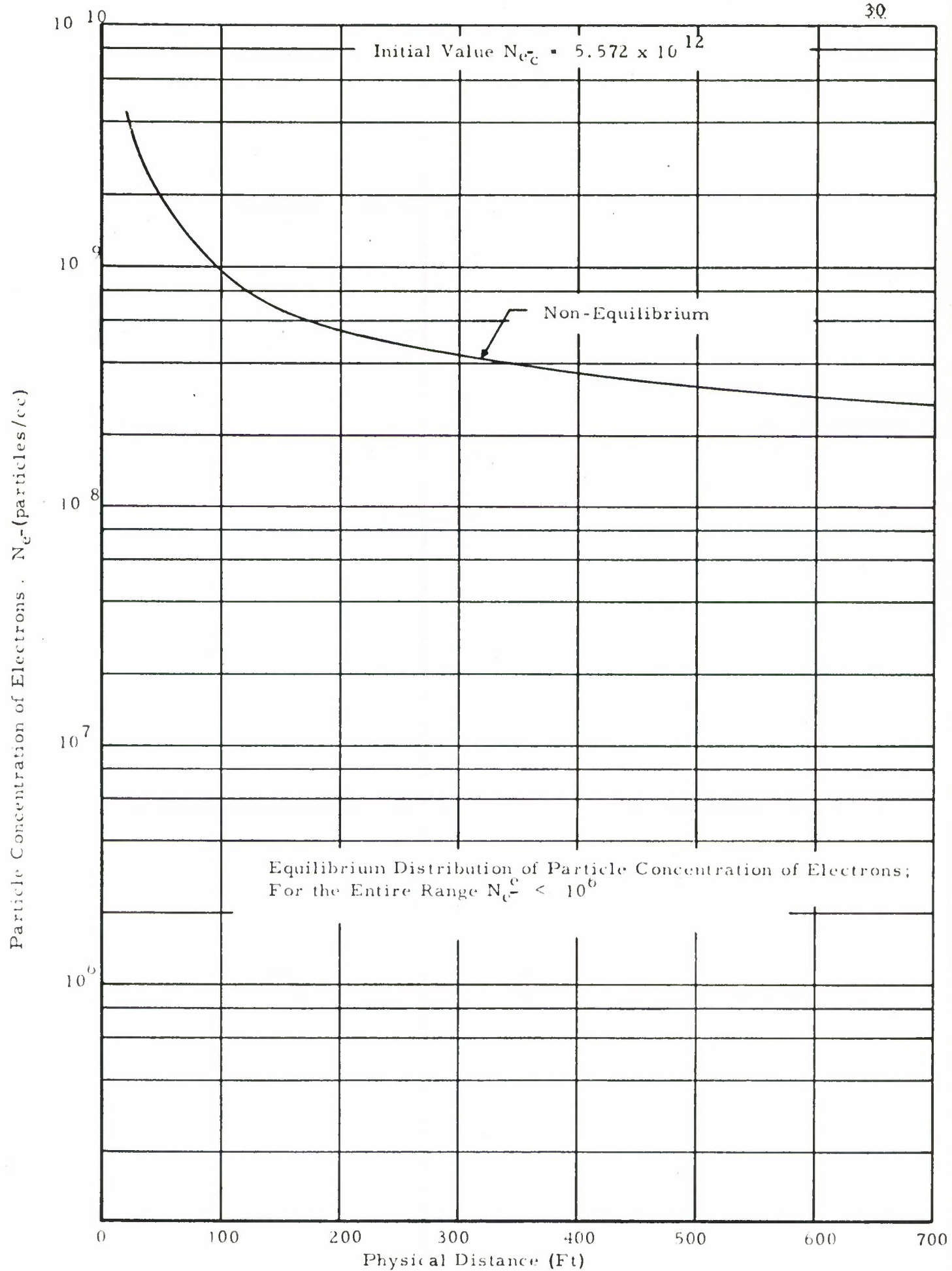


Figure 1d. Particle Concentration of Electrons Along the Axis